$A(x) = A_z = \frac{\mu}{4\pi} I_o \Delta l \frac{e^{-j\beta r}}{r}$ → Cartesian Coordinates (x,y,z)

We must change them Coordinates into spherical Coordinate
$(x, y, z) \rightarrow (r, \theta, \phi)$

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

$A_x = 0 \quad , A_y = 0 \quad A_z = \ldots$

$A_r = A_z \cos\theta \ \hat{r}$ $\Rightarrow A_r = \frac{\mu}{4\pi} I_o \Delta l \frac{e^{-j\beta r}}{r} \cos\theta \ \hat{r}$

$A_\theta = -A_z \sin\theta \ \hat{\theta}$ $\Rightarrow A_\theta = \frac{-\mu}{4\pi} I_o \Delta l \frac{e^{-j\beta r}}{r} \sin\theta \ \hat{\theta}$

$A_\phi = 0$

* Find $\bar{H}$ for infinitesimal dipole

$\bar{H} = \frac{1}{\mu} (\nabla \times \bar{A})$

$$\bar{H} = \frac{1}{\mu} \frac{1}{r^2 \sin\theta} \begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\, A_\phi \end{vmatrix}$$

$\downarrow$

$\left.\begin{matrix} H_r \\ H_\theta \\ H_\phi \end{matrix}\right\}$???

$\because A_\varphi = 0 \rightarrow \frac{\partial}{\partial \varphi} = 0$

(1) $H_r = 0$

(2) $H_\theta = 0$

(3) $H_\varphi = \frac{1}{\mu} \frac{1}{r^2 \sin\theta} \left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial}{\partial \theta}(A_r) \right] \times r \sin\theta \, \hat{\varphi}$

$$\frac{\partial}{\partial r}(r A_\theta) = \frac{\partial}{\partial r}\left[\cancel{r} \frac{\mu}{4\pi} I_o \Delta l \frac{e^{-JBr}}{\cancel{r}} \sin\theta \right]$$

$$= \frac{JB\mu}{4\pi} I_o \Delta l \, e^{-JBr} \sin\theta$$

$$\frac{\partial}{\partial \theta}(A_r) = \frac{\partial}{\partial \theta}\left[\frac{\mu}{4\pi} I_o \Delta l \frac{e^{-JBr}}{r} \cos\theta \right]$$

$$= \frac{-\mu}{4\pi} I_o \Delta l \frac{e^{-JBr}}{r} \sin\theta$$

$$H_\varphi = \frac{1}{\mu r}\left[\frac{JB\mu}{4\pi} I_o \Delta l \, e^{-JBr} \sin\theta + \frac{\mu}{4\pi} I_o \Delta l \frac{e^{-JBr}}{r} \sin\theta \right] \hat{\varphi}$$

$$= \frac{1}{\cancel{\mu} r} \frac{\cancel{\mu}}{4\pi} I_o \Delta l \, e^{-JBr} \sin\theta \left[JB + \frac{1}{r} \right] \hat{\varphi} * \frac{B^2}{B^2}$$

اوزع r ≠ اوزع = $\frac{B}{B^2}$

$$\boxed{H_\varphi = \frac{I_o \Delta l}{4\pi} e^{-JBr} \sin\theta \, B^2 \left[\frac{J}{Br} + \frac{1}{B^2 r^2} \right] \hat{\varphi}} \; \# \Rightarrow (1)$$

* لو جه في الامتحان هيحسب H_φ فقط دليل A_φ لانه
مطلوبة

Find $\bar{E}$ in infinitesimal dipole

$$\nabla \times \bar{H} = \frac{\partial \bar{D}}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t} \longrightarrow \frac{\partial}{\partial t} = J\omega$$

$$\boxed{\nabla \times \bar{H} = J\omega \epsilon \bar{E}}$$

$$\bar{E} = \frac{1}{J\omega \epsilon}\left[\nabla \times \bar{H}\right]$$

$$E = \frac{1}{J\omega \epsilon}\frac{1}{r^2 \sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & r\sin\theta\,\hat{\varphi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \varphi} \\ H_r & rH_\theta & r\sin\theta H_\varphi \end{vmatrix}$$

$$E_r = \frac{\partial}{\partial \theta}\left[r\sin\theta H_\varphi\right]\hat{r}$$

$$E_r = \frac{\partial}{\partial \theta}\left[r\sin\theta \frac{I_0 \Delta l}{4\pi} e^{-JBr} \sin\theta B^2 \left[\frac{J}{Br} + \frac{1}{B^2 r^2}\right]\right]\hat{r} \quad \sin^2\theta$$

$$* E_r = \left[\frac{I_0 \Delta l}{4\pi} r e^{-JBr} B^2\, 2\sin\theta \cos\theta \left[\frac{J}{Br} + \frac{1}{B^2 r^2}\right]\hat{r}\right] * \frac{1}{r^2 \sin\theta} * \frac{1}{J\omega\epsilon}$$

$$E_r = \frac{1}{J\omega \epsilon}\frac{2 I_0 \Delta l}{4\pi} B^2 e^{-JBr}\left[\frac{J}{B} + \frac{1}{B^2 r}\right]\cos\theta \frac{1}{r^2}\hat{r}$$

$$\omega\mu = \eta B \quad , \eta = \qquad B = \omega\sqrt{\mu\epsilon}$$

* [illegible]

(1) $H_\phi = \frac{I_0 \Delta l}{4\pi} \beta^2 e^{-j\beta r} \left[\frac{j}{\beta r} + \frac{1}{\beta^2 r^2} \right] \sin\theta \, \hat{\phi}$

(2) $E_r = \frac{I_0 \Delta l}{4\pi} \left(2 \omega \mu \beta \right) e^{-j\beta r} \left[\frac{1}{\beta^2 r^2} + \frac{1}{j \beta^3 r^3} \right] \cos\theta \, \hat{r}$

(3) $E_\theta = \frac{I_0 \Delta l}{4\pi} (\omega \mu \beta) e^{-j\beta r} \left[\frac{j}{\beta r} + \frac{1}{\beta^2 r^2} + \frac{1}{j \beta^3 r^3} \right] \sin\theta \, \hat{\theta}$

(4) $E_\phi = 0$

(5) $H_r = 0$

(6) $H_\theta = 0$

$$\boxed{I(z) = I_0 \operatorname{rect}\left(\frac{z}{\Delta l} \right)}$$

* for far field approximation

$\boxed{\frac{1}{r^2} = 0}$, $\boxed{\frac{1}{r^3} = 0}$

(1) $I(z) = I_0 \operatorname{rect}\left(\frac{z}{\Delta l} \right)$

(1) $H_\phi = \frac{I_0 \Delta l}{4\pi} \beta^2 e^{-j\beta r} \sin\theta \frac{j}{\beta r}$

$$\boxed{H_\phi = j\beta \frac{I_0 \Delta l}{4\pi} \frac{e^{-j\beta r}}{r} \sin\theta \, \hat{\phi}}$$

(2) $H_r = H_\theta = 0$

(3) $E_r = 0$

[4] $E_\theta = \frac{I_o \Delta l}{4\pi} \omega \mu \beta \, e^{-J\beta r} * \frac{J}{\beta r} \sin\theta \, \hat{\theta}$ //

$$\boxed{E_\theta = J\omega\mu \frac{I_o \Delta l}{4\pi} \frac{e^{-J\beta r}}{r} \sin\theta \, \hat{\theta}}$$ #

*

① Given $I(z)$

[2] $A_z = \frac{\mu}{4\pi} \frac{e^{-J\beta r}}{r} \, F.T \boxed{I(z)}$

$$\boxed{A_z = \frac{\mu}{4\pi} I_o \Delta l \frac{e^{-J\beta r}}{r}}$$

[3] $\boxed{E_\theta = J\omega A_z \sin\theta}$

[4] $\boxed{H_\phi = \frac{1}{\eta} E_\theta} \Rightarrow \frac{H_\phi}{E_\theta} = \frac{1}{\eta}$

$$\frac{\beta}{\omega\mu} = \frac{1}{\eta}$$

$$\eta = \frac{\omega\mu}{\beta}$$

حفظ لحل المسائل

* Antenna Parameters

لابد الحصول على E و H

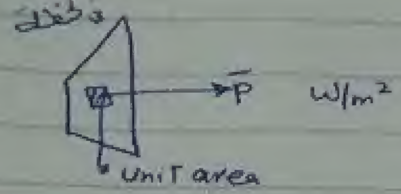
① Poynting Vector $\bar{P}$

$\bar{P} = \bar{E} \times \bar{H}$ $\quad W/m^2$

$\bar{E} \rightarrow V$, $\bar{H} \rightarrow I$

is The instantaneous Power flow Per unit area)

الطاقة التي تعبرها خلال وحدة المساحة



[2] Average radiated Power density $\overline{P_{av}}$

$$P_{av} = \frac{1}{2} R_e [\bar{E} \times H^*] \qquad W/m^2$$

(is the $\boxed{\text{Time averaged}}$ Power flow per unit area)

المتوسط خلال فترة زمنية